

*A relationship between lacanian theory of sexuation and brouwerian intuitionism*

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*Some introductory remarks*

An improbable interest in the question is inseparable from the two significant – Brouwer and Lacan – of the problem. The first justifies the question given the barbarous use of the master's thought by Lacanian schools, great and small, each dedicated to the lost cause of authenticity and orthodoxy. Our interest, however, does not lie there. Let us leave up to the professors of Lacanism the cares of demonstrating what Lacan really said. We prefer to examine how the work of this unique analyst fits into the European cultural context.

In this respect, the second significant - brouwerian intuitionism - is all the more attention-grabbing for reasons related not only to history but also to structure; that is, the pattern of how Lacan's theories differ from and are similar to others in fields contiguous to psychoanalysis. One of these is what is known by the unfortunate name of intuitionism, axiomatized in successive stages by Heyting, Kleene and Gentzen, and semantized by Jaskowski and Kripke. Without getting too technical, on this subject it must be said that with the weakening of the ideological bite of the contrast between Hilbert's formalism and Brouwer's intuitionism, thanks to the Gödel's incompleteness theorem, intuitionism revealed its true colours: not so much the logic of the mathematical act, the creative energy *ex nihilo* of which Brouwer confronted with the timeless Platonic essence of mathematical objects, but a simple mathematics among the many, easily recognised by its style. In fact, it is a particular form of mathematics that wants, really wants a lot, to weaken the binary game of true and false (understood in the classical sense as the involuted image of one another); therefore, it is more suited than other forms to mathematizing mathematical knowledge itself (and, in our opinion, analytical knowledge as well) with respect its own object, that is, the infinite, the mathematical object *par excellence*.

Intuitionistic mathematics in the tradition of Brouwer is today particularly alive. We shall mention just a few examples that are important for us as they occurred in a decade that was important and fertile for Lacanian thought. In 1958 Gödel showed the coherency of intuitionistic arithmetic of the first order (from a finitist point of view).<sup>1</sup> In 1962 Spector extended the result to intuitionistic analysis, an outstanding achievement unthinkable in classical mathematics.<sup>2</sup> In 1969 Fitting incorporated the idea of forcing into intuitionism, simplifying the arsenal Cohen used to create generic objects and demonstrate the independence of Cantor's continuum hypothesis from the rest of the theory of sets.<sup>3</sup> All this goes to show that once separated from the ideological premise that perceives mathematics as an alinguistic activity, presumed to have a direct link to the real, intuitionism has something worthwhile to say to whoever works with foundational problems of infinite. Of course as psychoanalysts our interest in this logic arises from the problems Freud put down in his Finite and infinite analysis.

In any case, since we are not professional mathematicians, we shall not discuss in general terms the question of intuitionism, for example as the basis for a not necessarily scientific epistemology of psychoanalysis (to make ourselves clear: not in the manner of Popper), but, with reference to the unique intellectual evolution of a particular subject, Dr. Lacan, who arrived at the holy threshold of intuitionism without ever decisively crossing it. But our job is not to be of an historical or philological nature. The honing of a thesis that

for now is only for fun and, in scientific terms, a bit farfetched, may be put off to a later date, after it has been demonstrated that if indeed Lacan was an intuitionist, he was unconsciously so, in the sense of the word that he gave to it, that is, without knowing that he knew it.

In effect, at first glance, it would seem that with his theory of the subject of desire as an ephemeral effect of language, Lacan is far from the act that according to Brouwer in his convoluted way of expressing himself, is the basic intuition of mathematics, inasmuch as it is an extralinguistic activity that perceives duty as a crucial moment of separation, and once it has been stripped of all its qualities it conserves the memory of its empty shape. In any case, all the typical Lacanian formulae of binary weakening, mostly existential negatives, from the there is not metalanguage to the there is no sexual relation, testify to a clear epistemic stance of intuitionist stamp. An act, that is, that if not a free choice in Brouwer's sense of the word, is one that perceives the essence of intuitionist thought as an attempt to overcome the binary regime.<sup>4</sup>

Though well aware that metaphors are usually confusing, we shall use one them less to make the concept clear than to link the sense of Lacan's performance to one that was perhaps even more decisive for psychoanalysis: that of Freud himself. A Jewish science, it was once said of psychoanalysis, and is still today whispered. A Jewish science, yes, but not only because Freud was a Jew; it may be a Jewish science because Freud tackled the question that Judaism alone among the book religions, managed to keep in focus without yielding to the temptation to simplify, as in Catholicism, or make abstract, as in Islamism: the question of the father. Thus, Lacan will be proven intuitionist less because he studied Brouwer in Berlin and more because he followed an ongoing tradition of thought about the function of knowledge, which can be traced back the Stoics and passes through Descartes and Spinoza, while still conserving its unique qualities that kept it distinct from ontological and hermeneutic thought, to arrive at the Vienna that was the home of Wittgenstein and Freud.

The very fact that he takes a precise epistemic stance (and whoever works with unconscious knowledge cannot help but have one) forces the analyst, whether he likes it or not, whether he is aware of it or not, to interact more or less closely with intuitionism. That is to say, at least he takes his distance from classical logic and its binary mode, which both make poor hosts to epistemic thinking. In its turn, epistemic thinking easily overflows the scheme of black and white certainties, taking on all sorts of intermediary forms that range from knowledge to ignorance, with in between all the epistemic combinations of operators of certainty and doubt, offering the certainty of doubt, the doubt of certainty, the doubt of doubt, the certainty of certainty, not to mention the ignorance of doubt and the doubt of ignorance and so forth.

The intermediary forms, instead, are well-suited to a logical setting, such as intuitionism, where above all, the shelving of the law of excluded middle clears the field of any naive, not to say contradictory, notion of omniscience in anyone – from the other to God – who would be able to decide on each separate occasion whether a pronouncement or its negation is true. By suspending the true/false dichotomy, intuitionism, in fact, leaves room for less totalitarian and more realistic epistemic hypotheses.

In the second place, in the intuitionist system the intolerance inherent to the binary regime – where to assume a thesis means to produce either theorems or contradictions – does not hold sway. Instead, its characteristic syntactic incompleteness is a trait that in our theoretical work we like to consider a preliminary condition for a soft epistemology of unconscious knowledge. The limitless possibilities for enriching it with new axioms

without necessarily introducing contradiction permit a conception of intuitionistic models of the unconscious as open systems of epistemic operators (each of whom applies all of the well-formed formulae in some classic, non-intuitionistic theses<sup>5</sup>) – as systems, that is, that can never be taken in hand all at once and classified once and for all in a few volumes in an encyclopaedia. But, as mentioned, we shall not go any further into this interesting point, which concerns the irreducibly impredicative, asymmetrical and intransitive nature of the unconscious, which thus appears more like a proper class (that is, not an element of another class) than a set (which, instead, is an element of another set).

Instead, we shall stress a detail that is interesting for what follows, and that is that in an intuitionist context negation – classically conceived as falsification inasmuch as it transforms the truth into falsehood and the falsehood into truth – takes on a special meaning that is quite close to the analytical one according to which falsehood is not the only way to negate the truth. In fact, in true analysis, the truth may be denied, rejected, repressed, etc...., while still remaining undeniably true. Correspondingly, again in analysis, knowledge cannot be negated even when expressed in negated, that is, unconscious, form, or as not knowing that one knows. In an intuitionist context, to negate – over and above being an oscillation, reversible but short-lived, between truth and falsehood, between two positions situation at the same epistemic level – forces the subject to step up to a higher level of logic where a full demonstration of the negation is needed. In a certain sense, reintroducing metalogic into logic, Brouwer suspends in absurdity the epistemic nucleus, or the content of knowledge, of negation. It is obvious that within such a theoretical approach, the universal validity of the law of excluded middle becomes untenable; for example, in infinite universes. What is less obvious is that it could contribute to clearing up the question of sexuation, where the exclusion of the middle leads to the paradoxical result that there are no longer two sexes to be counted, but if anything something less, though not necessarily reduced to just one, as, starting with oneness of the phallic representative, homosexuality, especially male homosexuality, tends to believe.

Here our question takes on a possible meaning. How did Lacan arrive at, if he arrived at, intuitionism? Certainly not by departing from questions of the epistemological foundation of analysis, but from a more banal issue, though one that is also spicier and less avoidable for the subject, that of sexuation. There are not two sexes, Lacan seemed to say in his conference of 3 March 1972, in the cycle entitled *Le savoir du psychanalyste*, held at Santa Anna contemporaneously with the *...ou pire Seminar*, (a paper from which, unfortunately as yet unpublished, we shall be discussing here); it was a direct challenge to the author of a (now erstwhile) cult book entitled *The Second Sex*. At the moment in which language starts to work, there is no second sex. For the talking being there is only heterosexuality, that is, the possibility for some (not all) to love women, with all of the tragedy that this may bring.

The first logical step in this direction is to get negation out of its binary complementation rut. It was taken by Sigmund Freud since 1899 and formalised in 1925. Lacan took hardly more than a few steps further along the road opened up by Freud, as we shall see. At first, the idea was received in circles outside of analysis better than among Freud's pupils. In 1937, Johansson constructed a form of logic, today called minimal, whereby nothing is implied by any false statement.<sup>6</sup> In particular, non-contradiction is valid only as an abstract principle that does not exclude single local contradictions, as some scholars naively believe (the explicit reference is to *Matte Blanco*).<sup>7</sup> Does this imply that all and all the contrary may be alleged. Surely not. In Freud, falsehood, no less than the truth, implies the responsibility of the subject, just in a way different than the truth. In his well

known thesis set forth in the article entitled *Negation*, 1925, Freud maintains that the significant of negation does not negate the objective formula, but indicates the passage of the subjective formulation through the barrier of repression. It thus finds new links with an epistemic tradition that, with its roots in Stoicism, comes fully to light only with Spinoza's ethics. It is the patently ethical tradition that, perceiving the false as not known or known insufficiently, brings into play the subject's responsibility with respect to his own knowledge, extended, even, to his own ignorance. The subject is responsible for what he knows just in the same way as for what he does not know.

This idea is not foreign to Brouwer's intuitionism, which regards knowledge about truth as not effaceable and about falsehood as partially effaceable (not definitively acquired). Therefore, the corresponding logistic system accepts only partially the classical dictum. It transcribes the truth of negation ( $V\neg X$ ) as the falsity of the statement ( $FX$ ), as Aristotelian logic would have it, but it differs in that it transcribes the falsity of the negation ( $F\neg X$ ) as the truth of the statement ( $VX$ ), on the condition of cancelling all the knowledge about the effect of other falsifications possible in the same epistemic state (or world), which thus disappear from the deductive tree. Is it just a question of prudence in carrying out the proof? No, it is also a way to bring into play the subject's responsibility in choosing the right negation of the falsity to be transformed into truth. The result: the mathematical proof, carried out in an intuitionistic way, is no longer that mechanical procedure of applying stereotyped formulae that we learn at school, but it more or less always retains something of the performance, the creative act.

Lacan takes up this epistemic tradition, proposing to interpret in a Freudian way the function of the negation as that which in the formula goes beyond the formulation: a trait that negation inherits from the phallic function, which is in itself excessive and always leads to beyond the sexual difference, with its presumptuous hegemony over every act of signification even blocking the sexual relation itself. In the conference we are discussing, Lacan stated his program with a fair dose of naivety, in conformity with the flashy public figure he had cut out for himself. Clearly, if I use a formulation that from mathematics (this plural form is always a welcome sound!) has broken into the field of logic, this does not mean that I am using it in exactly the same way. The naivety consisted in imagining that he could use significant as he pleased, avoiding the effects of alienation. Coming from the one who taught subjugation of the subject to the significant, at the very least it brings a smile to one's lips. In applying Lacan to himself, it will not be difficult to prove that in his attempt at an improper use of mathematical significant, with the praiseworthy aim of getting at the truth, he only makes proper use of them; that is, he lets himself be properly used (as any good mathematician would).

## Conclusion

The Lacanian square is well known.<sup>8</sup> Half of it is intentionally drawn along Aristotelian lines, while the other half distinguishes it. Half? Which half? In deference to his rhetoric of the chiasmus, which since the very first elucubrations led him to favour twisted models where double inversions reigned supreme (see, for example, the symbolic and imaginary double inversions in the L-scheme drawn as a Möbius strip), the half that Lacan selects from the Aristotelian square lies on the diagonal of the two statements that Scholastics call contradictory: the particular negative and the universal affirmative.

In illustrating the two formulations, Lacan shows how they conform to the logical tradition of male thought, interpreted in modern terms in the modern theory of sets, which in reality are always only subsets, as a set is such only if something remains outside of it. Let us not forget, however, that this is not an abstract matter; it stands in relation to a special propositional function, the phallic function, understood less in Russelian terms than Oedipal. The first column of the table thus appears as follows:

Particular negative:  $\exists x . \neg \Phi x$

Universal affirmative:  $\forall x . \Phi x$ .

Translated into plain words, this means: all speaking beings ( $\forall x$ ) are arguments of the phallic function ( $\Phi x$ ), or as that precocious intellectual the baby sees it, all are endowed with phallus:  $\forall x . \Phi x$ . Freud's correction to the theory of infantile psychology is that at least one individual, in this case, the primal father of the horde, is not subject to the phallic function inasmuch as he negates it, that is, he goes beyond it. *There exists an individual who says no to the phallic function* ( $\exists x . \neg \Phi x$ ), explains Lacan in the annotated text. In Freud's mythical picture, the primal father overcomes the phallic function by possessing all the women and forcing the brothers to homosexuality. The same story, played out in imaginary terms, for example by the little boy at the resolution of the Oedipal complex, says with pathetic resignation: *all are castrated, except him, the father*.

At a symbolic level, castration is like any other subset. All its elements are part of it, except those that are not part of it. A truism warns us that to think of a set that contains everything and leaves nothing out leads to Cantor's antinomy of the total set. The exception that is left out, here the father, normalises the matter. At times, and not entirely wrongly, the Freudian myth of Oedipus is referred to as the incarnation of the normalising function of subjectivity. Let us make it clear here that we are talking about the male (or religious) norm, the one that makes up the bastions of civilisation and its discontent: all are equal with respect to an ideal point on the outside and to which, according to the Freudian mythology, all is sacrificed, starting with the satisfaction of drives: yesterday to the monarch, today to the Law.

And the other half of the square? What does it say that is new? Everything and nothing. Nothing because it consists in a duplicate of the first half. Everything because it is a particular transcription that does not exactly match the original. Nothing because the duplicate is, if you will, a chiasmus applied to itself. As such it remains one of the rhetorical figures so dear to Lacan. But this is not the point. The problem is that Lacan, we shall never know why, leaves the universal negative ( $\neg \exists x . \Phi x$ : there does not exist an x who is castrated) and the particular affirmative ( $\exists x . \Phi x$ : there does exist an x who is castrated) out of his discussion of sexualisation. In his own way hung up on the medieval question of

universals, Lacan probably also treats negation as a universal quantifier and, to preserve the equilibrium, instead of completing the Aristotelian square, he doubles the male rectangle in the female one (perhaps also to accommodate the Freudian error of a single libido, the male one). In effect, he obtains the second from the first in three moves: the first is the typically Lacanian inversion and the other two, we stress the fact, are quite anti-intuitionist: the double substitution of the *for everyone* with the *there does not exist one who is not* and the *for someone* with the *not for all does not* and the cancellation of the double negations thus introduced. The complete result may be read in the following table:

$\exists x. \neg \Phi x$	$\forall x. \Phi x$	$\neg \exists x. \neg \Phi x$ *	$\neg \exists x. \neg \Phi x$
$\forall x. \Phi x$	$\exists x. \neg \Phi x$	$\neg \forall x. \neg \neg \Phi x$ *	$\neg \forall x. \Phi x$ *

Here, the first column on the left represents the male side of sexuation, the last on the right the female and the middle ones the logical transition from one to the other. The non-intuitionist passages governed by the law of excluded middle are indicated with an asterisk.

At this point, our theory about Lacan intuitionist seems to be heavily compromised. The passage from male to female sexuation requires at all levels, universal and particular, the application of the law of excluded middle. Back to the drawing board, then? Maybe not. Whoever recognises in  $\neg$  the Bourbakian symbol for negation (in the sense it is false that), a cursory glance at the formulae – which immediately ought to appear self-contradictory not just in the vertical direction, as expected, but now also horizontally – should reject the idea that it is an extensional subset of the field of speakers. So is it an empty subset? Not exactly. Intensional, then? Before resorting to by now obsolete terms, already used by such idealists as Frege, for example, let us pause a moment for thought.

In a slightly later text, *L'étourdit* (Scilicet 4, Seuil, Paris, 1973), Lacan explicitly states: “the writing of formulae regarding female sexuation ( $\neg \exists x. \neg \Phi x$  e  $\neg \forall x. \Phi x$ ) is not usual in mathematics” (p. 22). This is not entirely true. In notes and observations, more than in the formulations of the theorems, the mathematician willingly yields to considerations on the non-existence of objects that do not satisfy given properties, or ones on the general invalidity of certain affirmations. On the other hand, Brouwer himself in 1949 proved a full continuum theorem formulated in terms of not for every: *Not for every x is it true that if x is different from zero, then either x is greater than zero or x is less than zero*. Is this the hoped-for entry in mathematics of female logical modes? Ask Sophie Germain, who corresponded with Carl Gauss on the theory of numbers (about a very special case in which Fermat’s Last Theorem is not valid) under the pseudonym M. Le Blanc, to avoid the ridicule inevitably associated with the condition of the woman scholar. And if the contrary were true? That female modes, being particularistic, were essentially antimathematical? There is no use saying it. Counterfactual matters are not the mathematician’s strong point. Therefore, we shall interrupt the argument here.

In his notes on his feminine formulae, Lacan introduces the notion of not for all as that which goes beyond the all, a universal more universal than the universal, where nothing is left out (the Latin *ex-sistere*) and which for this cannot be defined conceptually as an all. In effect, the not all is an all lacking a perimeter and extensional unit. It is given by the aggregate of its elements, the elementary units, but it itself is not reducible to an element of other universals. This would be the universal that befits the other sex, definitively

heterosexual because it remains forever other with respect to any possible conceptual premise, including that of castration. To assert that not all women are castrated ( $\neg \forall x. \Phi x$ ), though taken one by one there does not exist one who is not ( $\neg \exists x. \neg \Phi x$ ) is a paradox more apparent than real. It belongs to the series of *Paradoxes of the infinite* that Bolzano wrote about but did not publish. Students of logic, for example, are well aware that a theory may be infinitely consistent without being consistent. As far as femininity is concerned, where Lacan applies the not for all notion, all this means that castration, like any other quality, however forceful it is, however far it goes beyond collectivising qualities, as Bourbaki calls them (and castration is one of them), does not define femininity as a type, that is as a whole endowed with an own unit. As such (we should not say as such, because it implies unification, but nevertheless...), as such femininity cannot be reduced to any scheme of belonging or conformity to some ideal. Metaphorically speaking, femininity may contain but not be contained.

For this and other analogous reasons, civilisation willingly forces femininity into its schemes, primarily into the maternal one, because with its wild and laic universality, irreducible to a code or rite, it does not compromise the sacredness of its institutional containers which, as everybody knows, are heavily limited even when they are enlightened and liberal. Sophocles dramatised the relationship between the two universals, male and female, in the conflict between the characters Creon and Antigone. Today, more prosaically, the control of femininity, as long as it conforms to the social standard, is entrusted to psychotherapy. The results, to tell the truth, are questionable. Everything depends on how it will cope with anorexia. (But the analyst is cheering on the side of anorexia. Some psychotherapeutic slop truly is nauseating. Anorexia is a right... for all.)

At the level of abstract logic (abstract is not a naughty word), the contrast between the two forms of universals may be reduced to the alternative between proper classes that do not belong to classes and sets that belong to classes. The formalisation of set theory, where the abovementioned distinction is articulated, was first proposed by von Neumann (and even suggested by Cantor) and elaborated by Gödel and Bernays. From the viewpoint of sexuation, it is doubtlessly more satisfying that the acrobatics attempted by Lacan on the Freudian negation tightrope (however well supported by the balancing rod of the distinction between the two Greek particles of negation: *ou* for objective negation and *mé* for subjective). On the one hand, the classes themselves are the positive way of saying the not for all and to assert femininity in terms of what is not assertable about it. On the other, the sets are much more manageable and for centuries have been familiar territory for logicians and western politicians; with their bent for the transcendental (towards the exterior), barely masked by a semblance of internal democracy, they will continue to embody the male. (The Christian democratic history of the first Italian republic is an illuminating example in this respect.) We must note however that the contrast between the two modes of universalization does not exist, because when they make a couple, it would give rise to a case of a proper class making up the element of a couple, that is of a set, which is contrary to the spirit of its definition.

Let us rapidly conclude.

What relation can be established between the two columns, right and left, of the Lacanian square, where each term represents the negation of the homologous term on the same line, exactly as in *A* or *not A*. A sexual relation seems to be excluded, since the two columns are the same sex in different guise. In the conference we have been discussing, Lacan ventured into considerations of symbolic logic, bolstered by a few notions he had

garnered from some manual. And, with the courage provided by his ignorance, he dared to assert that between the right and the left of the table, as between mirror-image right and left, there can be no relation at all, either of conjunction or of disjunction, or, finally of implication.

Hold your fire, people. Lacan's assertion is valid for conjunction and implication. In classical, or, rather, binary logic, the conjunction of  $A$  and  $not-A$ , like the implication of  $A$  by  $not-A$ , are not logical theses. But the disjunction is.  $A$  or  $not-A$ , is a famous, classical thesis, quite valid. It is the well known law of excluded middle. So, what does Lacan mean when he asserts that in his square disjunction cannot apply? The answer, all said and done, is fairly predictable. It means that the Lacanian square is not Aristotlean but at most intuitionist. In fact, only in intuitionist or weaker logic is it not generally accepted as given that  $A$  or  $not-A$  is true.

Support for the argument that Lacan always had intuitionist leanings, despite passing transgressions (such as those mentioned above, which were not the only ones) may be found ex post throughout the man's intellectual career. For what its worth, we may mention just one representative case: his three-dimensional concept of the symbolic register, which brings the three-dimensionality into play twice.

The symbolic register is the intermediary between the two protagonists of the narcissistic relationship, the ego and its mirror-image. As the intermediary, the symbolic bears the paternal law that separates the mother from son, the law that the lads have interpreted as the law of castration. But the symbolic register is intermediary even with respect to itself. In fact, as a linguistic register, it is made up of the contrast of elements, the significant. The mother of every significant contrast is, however, the binary articulation of presence and absence. This is as far as classical logic goes and where its latest incarnation, informatics, joins up. Lacanian intuitionism goes one step further. It sets presence-absence within the context of the roll call (see Seminar IV).<sup>9</sup> The roll call summons the intermediary to coordinate in itself absence (of the object) and presence (of the subject). If it does not fail to answer at the roll call of the Other, the intermediary's action will be expressed in a typical negative effect: the absence of the object is inverted into want, which precedes every loss, and the presence of the subject is transformed into evanescent matter, a subversion that beats in time any form of nihilism. Thus, the symbolic sustains the vagaries of matters of drive which, the less biologically and the more linguistically structured they prove to be, the stranger they sound to non-analytic ears.

But it is not the case to go any further. It suffices to have pointed out a way to reinterpret the laic nature of analysis as inclusion in its logic of the function of the third: a way implemented in mathematics by Brouwer and in analysis by Lacan.

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